

From the Komar mass and entropic force scenarios to the Einstein field equations on the AdS brane

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By bearing the Komar's definition for the mass, together with the entropic origin of gravity in mind, we find the Einstein field equations in $(n + 1)$ -dimensional spacetime. Then, by reflecting the $(4 + 1)$ -dimensional Einstein equations on the $(3 + 1)$ -hypersurface, we get the Einstein equations onto the 3-brane. The corresponding energy conditions are also addressed. Since the higher dimensional considerations modify the Einstein field equations in the $(3 + 1)$ -dimensions and thus the energy-momentum tensor, we get a relation for the Komar mass on the brane. In addition, the strongness of this relation compared with existing definition for the Komar mass on the brane is addressed.

I. INTRODUCTION

The discovery of the thermodynamic roots of gravity comes back to the works of Bekenstein and Hawking [1–3]. These attempts were followed [4–7] and finally formed the backbone of the well-known paper by Jacobson [8]. Indeed, Jacobson has considered the same relation between the horizon entropy and its surface area as the Bekenstein bound [1], and showed that for the static spacetimes the Einstein field equations on the horizon is equal to the thermodynamic identity $\delta Q = T\delta S$. This setup were also extended to $f(R)$ gravity [9]. In fact, by using the first law of thermodynamics on the horizon, it was proved that the gravitational field equations can be rebuilt in a wide range of theories [10]. The same deductions for cosmological setups [11–18] and braneworld scenarios [19–24] are valid. A comprehensive review can be found in Ref. [25].

Thermodynamic aspects of the gravity can help us to have a better understanding of the nature of spacetime and the gravity. Padmanabhan claimed that spacetime includes unknown structure in the microscopic scales inducing the degrees of freedom for the spacetime, and their statistical description yields the gravity [26]. Another parallel approach suggested by Verlinde [27]. He showed that the tendency of systems to increase their entropy may lead to the emergence of the gravity. Therefore, gravity is not a fundamental force and can be considered as a secondary effect. This approach is called entropic force and has attracted a lot of investigations [28–48].

String theory, as a promising approach to quantum gravity, predicts eleven dimensions for the spacetime [49]. The AdS/CFT correspondence conjecture relates an n -dimensional conformal field theory to an $(n + 1)$ -dimensional gravity theory in an anti-de Sitter (AdS) space [50, 51]. Indeed, such generalization to the n -dimensions is due to the holographic principle [52, 53]. Based on these motivations, bearing the Komar's n -dimensional definition of mass [28, 29] in mind and using the entropic force approach, the Friedmann equations of the $(n + 1)$ -dimensional gravity theories including the Einstein, Gauss-Bonnet and Lovelock theories are obtainable [48].

Another striking motivation for studying the higher dimensional gravity comes from braneworld scenarios. In these scenarios there is a $(4 + 1)$ -dimensional spacetime and unlike other fields, gravity can propagate in the fifth dimension. It was argued that the braneworld scenarios can explain the weakness of the gravity compared with the other fundamental forces [54, 55]. By taking the entropic origin of the gravity into account, authors of [42] tried to find the Friedmann equations on the brane. In addition, author of [43] has considered corrected entropy of the horizon and found the modified Friedmann equations on the brane from the entropic force. In these approaches [42, 43], authors have introduced a same definition for the Komar mass on the brane. In this paper, based on the Komar definition for the mass in n -dimensions, we try to build the $(n + 1)$ -dimensional Einstein field equations using the entropic force approach. In continue we point to this fact that by following the covariant approach of the authors of [56], one can reach to the Einstein field equations onto the brane. We also study the validity of some energy conditions on the brane. We also get a relation for the Komar mass on the brane which is in accordance with the higher dimensional modifications to the Einstein equations onto the brane. In addition, we compare our result for the Komar mass on the brane with the previous definition introduced in [42, 43]. Finally, we point to weaknesses of the Einstein field equations derived by using the entropic force scenario together with the Komar mass definition used in [42, 43].

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II. GRAVITY IN $(n + 1)$ -DIMENSIONS

According to the Verlinde's proposal, the tendency of the systems to increase their entropy leads to the emergence of gravity [27]

$$F = T \frac{\Delta S}{\Delta x}. \quad (1)$$

In this approach there is a surface sphere (holographic screen) with radius r which encloses the source of energy. The holographic principle implies $S \sim A$, where A is the surface area of the holographic screen [47]. Since the gravitational information of the energy source is distributed over N bits on the holographic screen, we also have $S \sim N$ [47]. In addition, the entropy of the gravitational system, according to the Bekenstein argument [1], is given by

$$S = \frac{A}{4\ell_p^2}, \quad (2)$$

where ℓ_p is the Plank's length and A is the surface area of the three-dimensional holographic screen [27]. Since the possible maximum number of the bits on the three dimensional holographic screen is given by

$$N = \frac{A}{\ell_p^2}, \quad (3)$$

we reach $S = \frac{N}{4}$ for the relation between the entropy and the number of the bits on the three dimensional holographic surface [27, 47]. The Unruh temperature associated with the holographic screen can be written [6],

$$k_B T = \frac{\hbar}{2\pi c} a, \quad (4)$$

where a is the acceleration of the Unruh observer. One can generalize this temperature to every accelerated observer when $F \neq 0$ [27]. In Eq. (1), Δx is the displacement of the test mass (m) from the holographic screen. When Δx , is of order of Compton wavelength $\lambda_m = \frac{\hbar}{mc}$, the test mass will be absorbed by the source [27]. In order to generalize our study to the arbitrary dimensions, we consider an $(n + 1)$ -dimensional spacetime with timelike Killing vector ξ^α , where its metric is $g_{\mu\nu}$. So, $\phi = -\frac{1}{2} \log \xi^\alpha \xi_\alpha$ is the generalization of the Newton's potential [27]. The mass M induces a holographic screen Σ_n at distance r and for the volume and the area of this n -sphere, we have

$$V_n = \Omega_n R^n, \quad \Sigma_n = n\Omega_n R^{n-1}, \quad (5)$$

where

$$\Omega_n = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2} + 1)}, \quad \Gamma(\frac{n}{2}) = (\frac{n}{2} - 1)!. \quad (6)$$

The $(n + 1)$ -dimensional gravitation constant may be written as [47, 48]

$$G_{n+1} = 2\pi^{1-n/2} \Gamma\left(\frac{n}{2}\right) \frac{c^3 \ell_p^{n-1}}{\hbar}. \quad (7)$$

Generalization of (3) to n -dimension yields [47, 57]

$$\Sigma_n = N \ell_p^{n-1}, \quad (8)$$

while we still have

$$S \sim \Sigma_n, \quad (9)$$

which is due to the holographic principle [47, 57]. Consider the total energy of the system as

$$E = Mc^2, \quad (10)$$

which is regularly distributed over the N bits. According to the equipartition law of energy we have [58]

$$E = \frac{1}{2} N k_B T. \quad (11)$$

By combining this equation with (10) and employing the general equipartition law of the energy, we arrive at [27]

$$M = \frac{1}{2c^2} \int k_B T dN. \quad (12)$$

In the above equation T is the temperature of the holographic screen. Following Unruh's argument, as well as the relation between ϕ and a , we get [27]

$$k_B T = \frac{\hbar}{2\pi c} e^\phi N^b \nabla_b \phi, \quad (13)$$

where N^b is an outward pointing vector which is normal to both the screen Σ_n and ξ^α [27]. Inserting (13) into Eq. (12), we find

$$M = \frac{\hbar}{2\pi c^3} \int e^\phi N^b \nabla_b \phi dN. \quad (14)$$

Using the relation between ϕ and ξ^α , as well as Stokes theorem, we find [27, 59]

$$M = \frac{\hbar}{4\pi c^3 \ell_p^{n-1}} \int R_{\mu\nu} n^\mu \xi^\nu dV_n, \quad (15)$$

where n^μ is a normal to the volume V_n . Combining this result with Eq. (7) yields

$$M = \frac{\Gamma(\frac{n}{2})}{2\pi^{\frac{n}{2}} G_{n+1}} \int R_{\mu\nu} n^\mu \xi^\nu dV_n. \quad (16)$$

Therefore, the $M \geq 0$ condition is similar to the Averaged Strong energy condition (ASEC) [60]. For $n = 3$ this relation is reduced to

$$M = \frac{1}{4\pi G} \int R_{\mu\nu} n^\mu \xi^\nu dV_3, \quad (17)$$

where $G \equiv G_4 = c^3 \ell_p^2 / \hbar$, is the four-dimensional Newtonian gravitational constant and so, previous result is recovered [27, 59].

Nowadays, it is generally accepted that the active gravitational mass (Komar mass) plays the role of the mass [61]. The Komar mass in $(n+1)$ -dimensions is defined as [28, 29]

$$\mathcal{M} = \frac{n-1}{n-2} \int_{V_n} dV_n \left(T_{\mu\nu} - \frac{1}{n-1} T g_{\mu\nu} \right) n^\mu \xi^\nu. \quad (18)$$

It is useful to mention that if the right hand side of this equation is positive, then we face with positive energies ($\mathcal{M} \geq 0$) which is also similar to ASEC [60]. In this equation, $T_{\mu\nu}$ is the energy momentum tensor in $(n+1)$ -dimension. By equating (16) and (18) we get

$$R_{\mu\nu} = \frac{2(n-1)\pi^{\frac{n}{2}} G_{n+1}}{(n-2)\Gamma(\frac{n}{2})} \left(T_{\mu\nu} - \frac{1}{n-1} T g_{\mu\nu} \right). \quad (19)$$

So, the $(3+1)$ -dimensional Einstein equation will be restored by choosing $n = 3$. The relation between Einstein gravitational constant in $(n+1)$ -dimensions, κ_{n+1} , and the Newtonian constant G_{n+1} can be written as [48, 62]

$$\kappa_{n+1} = \frac{2(n-1)\pi^{n/2} G_{n+1}}{(n-2)(\frac{n}{2}-1)!}. \quad (20)$$

Substituting (20) in (19), we obtain

$$R_{\mu\nu} = \kappa_{n+1} \left(T_{\mu\nu} - \frac{1}{n-1} T g_{\mu\nu} \right), \quad (21)$$

which can also be rewritten as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa_{n+1} T_{\mu\nu}. \quad (22)$$

In this equation, $R_{\mu\nu}$ and $T_{\mu\nu}$ are Ricci tensor and energy momentum in $(n+1)$ -dimensions, respectively. For $n=3$ we have $\kappa_4 = 8\pi G$ and the well-known coefficient of the Einstein equation is restored [63]. Therefore, Eq. (22) is nothing but the $(n+1)$ -dimensional Einstein equations. Now, consider an $(4+1)$ -dimensional manifold which is described by the line element

$$ds^2 = d\zeta^2 + q_{\mu\nu}dx^\mu dx^\nu. \quad (23)$$

In the above equation, $q_{\mu\nu}$ is the metric of the $(3+1)$ -dimensional submanifold $\zeta = \text{constant}$. Without lose of generality, we can choose the hypersurface $\zeta = 0$. Hence, we have

$$g_{\mu\nu} = q_{\mu\nu} + n_\mu n_\nu, \quad (24)$$

where $n^\mu = \delta_\zeta^\mu$ is the spacelike unit normal vector to the $(3+1)$ -dimensional submanifold. We assume the energy momentum tensor of the $(4+1)$ -dimensional bulk has the following form [56]

$${}^5T_{\mu\nu} = -\Lambda g_{\mu\nu} + \delta(\zeta)S_{\mu\nu}. \quad (25)$$

In this equation, $\Lambda = -\frac{6}{\ell^2}$ is the bulk cosmological constant where ℓ is the curvature radii of the bulk. In addition, since for $r < \ell$ the effects of the extra dimension may have an acceptive contribution to the gravity compared with those of the ordinary dimensions, ℓ can be also considered as the effective size of the extra dimension [64]. In addition, $S_{\mu\nu}$ is decomposed into two parts including the energy momentum tensor of the ordinary matter ($\tau_{\mu\nu}$) and the tension (λ) of the 3-brane,

$$S_{\mu\nu} = \lambda q_{\mu\nu} + \tau_{\mu\nu}. \quad (26)$$

From this expression for the energy momentum tensor of the bulk, it is apparent that, unlike the other matter field, gravity can penetrate into the fifth dimension. We should note that M_p is the Planck mass and $\lambda = \frac{3M_p^2}{4\pi\ell^2}$. In addition, it seems that the fifth dimension may affect the Newton's law of gravity when the effective size of the extra dimension satisfies the $\ell \leq 0.1$ mm condition, and therefore one gets the $\Lambda \leq -6 \times 10^8 \text{eV}$ and $\lambda > (1 \text{ TeV})^4$ limits for the bulk cosmological constant and the brane tension respectively [64, 65]. This setup, which was originally proposed by Randall and Sundrum [55], can explain the weakness of the gravity against the other fundamental forces [55]. The key point in obtaining the Einstein equations on the brane is reflecting the $(4+1)$ -dimensional Einstein equations on the 3-brane, which is a mathematical problem [56]. In fact, the Einstein equations for the bulk can be written as

$${}^5G_{\mu\nu} \equiv {}^5R_{\mu\nu} - \frac{1}{2} {}^5R g_{\mu\nu} = \kappa_5 {}^5T_{\mu\nu}. \quad (27)$$

Also from (25), the Israel's junction conditions read [56]

$$[q_{\mu\nu}] = 0, \quad [K_{\mu\nu}] = -\kappa_5 \left(S_{\mu\nu} - \frac{S}{3} q_{\mu\nu} \right), \quad (28)$$

where $[A] \equiv \lim_{\zeta \rightarrow C^+} A - \lim_{\zeta \rightarrow C^-} A$. In order to find the projection of (27) on the brane, we multiply (27) by $q_\beta^\mu q_\delta^\nu$,

$${}^5G_{\mu\nu} q_\beta^\mu q_\delta^\nu = \kappa_5 {}^5T_{\mu\nu} q_\beta^\mu q_\delta^\nu. \quad (29)$$

Taking into account the Z_2 symmetry of the bulk, and using the relations between Einstein tensor in four and five dimensions as well as the Israel's junction conditions, and following the approach of the authors in [56], we arrive at

$${}^4G_{\beta\delta} = 8\pi G_N {}^4T_{\beta\delta}. \quad (30)$$

In deriving above equation, we have used the following definitions

$$\begin{aligned} {}^4T_{\beta\delta} &\equiv \tau'_{\beta\delta} + \frac{6}{\lambda} \Pi_{\beta\delta} - \frac{6}{\lambda \kappa_5^2} E_{\beta\delta}, \\ \tau'_{\beta\delta} &\equiv \tau_{\beta\delta} - Q q_{\beta\delta}, \end{aligned} \quad (31)$$

where

$$\Pi_{\beta\delta} \equiv -\frac{1}{4} \tau_{\beta\alpha} \tau_\delta^\alpha + \frac{1}{12} \tau \tau_{\beta\delta} + \frac{1}{8} q_{\beta\delta} \tau_{\mu\nu} \tau^{\mu\nu} - \frac{1}{24} q_{\beta\delta} \tau^2, \quad (32)$$

and

$$Q = \frac{3\Lambda}{\kappa_5\lambda} + \frac{\lambda}{2}, \quad G_N = \frac{\kappa_5^2\lambda}{48\pi}. \quad (33)$$

Indeed, $\Pi_{\mu\nu}$ is a geometrical term coming from the extrinsic curvature terms, which is written in the (32) form, by considering the Israel junction conditions along as the $(4+1)$ -dimensional Einstein equations [56, 66]. Also $E_{\beta\delta} = {}^5C_{\mu\nu\gamma}^\alpha n_\alpha n^\nu q_\beta^\mu q_\delta^\gamma$ is a traceless tensor including the projection of the five dimensional Weyl tensor (${}^5C_{\mu\nu\gamma}^\alpha$) onto the brane meaning that $E_{\beta\delta}n^\beta = 0$ [67]. In addition, Q and G_N are the vacuum energy density and the Newton's gravitational constant on the brane respectively. Therefore, in this way we obtain the Einstein equations onto the brane by reflecting the $(4+1)$ -dimensional Einstein field equations on the $(3+1)$ -dimensional hypersurface. In addition, $E_{\beta\delta}$ is the limiting value at either $\zeta \rightarrow 0^+$ or $\zeta \rightarrow 0^-$. It is worth to mention that G_N differs from the ordinary Newtonian gravitational constant in $(3+1)$ -dimensional spacetime (7). Also, the relation between G_N and κ_5 differs from (20). In addition, for $\lambda \leq 0$, we see that the Newtonian gravitational constant on the brane is misdefined. In fact, these differences originates from the projective nature of the Einstein equations onto the brane. Finally, we should note that Eq. (30) converges to the general relativity, provided we neglect the bulk effects, namely we take the $\kappa_5 \rightarrow 0$ limit while keeping G_N finite [56].

Bianchi identity implies $D_\beta {}^4G^{\beta\delta} = 0$, where D_β denotes the covariant derivative in 4-dimensional spacetime, which leads to

$$D_\beta {}^4T^{\beta\delta} = 0. \quad (34)$$

Whenever the ordinary matter fields $(\tau_{\beta\delta})$ are only distributed on the brane (25), since $D_\mu g^{\mu\nu} = 0$, we get $D_\beta \tau^{\beta\delta} = 0$ and

$$D_\beta E^{\beta\delta} = \kappa_5^2 D_\beta \Pi^{\beta\delta}. \quad (35)$$

Indeed, Eqs. (34) and (35) are nothing but the conservation equations on the brane [56, 68]. It is useful to note that for a pure anti de-Sitter bulk we reach

$$D_\beta \Pi^{\beta\delta} = 0, \quad (36)$$

since $E_{\beta\delta} = 0$ [68]. The energy-momentum tensor of a perfect fluid source is seen by an observer with four velocity u^ν as

$$\tau_\mu^\nu = (\rho + p)u_\mu u^\nu + p\delta_\mu^\nu, \quad (37)$$

leading to (32)

$$\Pi_\mu^\nu = \frac{\rho}{6}[(\rho + p)u_\mu u^\nu + (\frac{\rho}{2} + p)\delta_\mu^\nu], \quad (38)$$

where we have considered the $(- + + +)$ signature for the brane metric. Using Eq. (36), it is easy to show that an inhomogeneous perfect fluid is rejected whenever, the bulk is purely anti de-Sitter [67, 68]. For any non-spacelike observer, which moves onto the brane, with four velocity u^ν , the energy density should be positive which is called the weak energy condition (WEC) [60, 69]. Since $\tau_{\mu\nu}$ carries the information of energy source, WEC implies that $\tau_{\mu\nu}u^\mu u^\nu \geq 0$ leading to

$$\rho \geq 0, \quad \rho + p > 0, \quad (39)$$

where we have used (37) to get this relation [69, 70]. Applying WEC to Π_μ^ν introduced in (38), we get

$$\frac{\rho^2}{12} \geq 0, \quad \frac{\rho}{6}(\rho + p) > 0. \quad (40)$$

It is apparent that if WEC is satisfied by $\tau_{\mu\nu}$, then $\Pi_{\mu\nu}$ will also satisfy WEC. It is straightforward to see that the $-Qg_{\mu\nu}$ term satisfies $\rho \geq 0$ for $Q > 0$, and does not satisfy $\rho + p > 0$ ($\rho + p = Q - Q = 0$). Therefore, WEC is not fully satisfied by $-Qg_{\mu\nu}$. Additionally, WEC is completely violated by $-Qg_{\mu\nu}$ while $Q < 0$. From geometrical point of view, WEC implies that ${}^4G_{\mu\nu}u^\mu u^\nu \geq 0$ which leads to the ${}^4T_{\mu\nu}u^\mu u^\nu \geq 0$ condition [60]. It is also useful to mention that WEC may be violated by the $E_{\mu\nu}$ term [71]. The latter may lead to ${}^4T_{\mu\nu}u^\mu u^\nu < 0$ telling us that WEC can be violated by the ${}^4T_{\mu\nu}$ term, independent of $-Qg_{\mu\nu}$ [71]. Moreover, since a null observer is a non-spacelike observer, by continuity we can conclude that the energy density corresponding to a null observer, with tangent vector field k^ν ,

should be positive meaning that $\tau_{\mu\nu}k^\mu k^\nu \geq 0$ [59, 60, 69, 70]. ${}^4G_{\mu\nu}k^\mu k^\nu \geq 0$ is the geometrical interpretation of this energy condition leading to ${}^4T_{\mu\nu}k^\mu k^\nu \geq 0$ [60]. By applying this condition to (37), we get

$$\rho \geq 0, \quad \rho + p \geq 0. \quad (41)$$

Indeed, WEC implies NEC [59, 60, 69, 70]. For the $\Pi_{\mu\nu}$ term, we reach

$$\frac{\rho^2}{12} \geq 0, \quad \frac{\rho}{6}(\rho + p) \geq 0, \quad (42)$$

where we have used (38) to obtain this equation. Therefore, if NEC is satisfied by $\tau_{\mu\nu}$, then NEC is also satisfied by $\Pi_{\mu\nu}$. Moreover, NEC is marginally satisfied by the $-Qg_{\mu\nu}$ term for $Q > 0$, and is not satisfied for $Q < 0$. As the WEC case, NEC may be violated by the $E_{\mu\nu}$ term and thus the ${}^4T_{\mu\nu}$ term [71].

Physically, Dominant Energy Condition (DEC) implies that the density of matter momentum ($-\tau^{\mu\nu}u_\mu$) measured by an observer with a four velocity u^μ should be non-spacelike [69, 70]. Applying this condition to (37), one gets $\rho \geq 0$ and $|p| \leq \rho$ [69]. Calculations for (38) leads to $\rho \geq 0$ and $|1 + \frac{2p}{\rho}| \leq 1$. The latter means that p should either satisfy the $p \leq 0$ or $-p \leq \rho$ conditions. Therefore, for $0 \leq p \leq \rho$, $\tau_{\mu\nu}$ and $\Pi_{\mu\nu}$ satisfy WEC, NEC and DEC simultaneously. Moreover, it is straightforward to see that the $-Qg_{\mu\nu}$ term will marginally satisfy DEC, only for $Q > 0$. From geometrical point of view, DEC implies that $-{}^4G^{\mu\nu}u_\mu$ should be causal [60]. By using this geometrical interpretation, it is shown that $E_{\mu\nu}$ may violate DEC which can lead to violate DEC by ${}^4G^{\mu\nu}$ and thus ${}^4T_{\mu\nu}$ [71].

In order to derive a suitable criterion for studying the Strong Energy Condition (SEC), we should write the Raychaudhuri equation for a congruence of timelike geodesics on the brane as [67]

$$\frac{d\Theta}{d\lambda} + \frac{1}{3}\Theta^2 + \sigma_{\mu\nu}\sigma^{\mu\nu} - \omega_{\mu\nu}\omega^{\mu\nu} + 8\pi G_N(\tau_{\mu\nu} - \frac{1}{2}\tau q_{\mu\nu})u^\mu u^\nu - 8\pi G_N Q = -\frac{1}{12}\kappa_5^2(\rho(2\rho + 3p)). \quad (43)$$

In deriving the above equation, we have assumed that $\tau_{\mu\nu}$ has the perfect fluid form, and did the calculations for a purely anti de-Sitter bulk meaning that $E_{\mu\nu} = 0$. These considerations lead to a homogeneous density and thus homogeneous pressure which implies that the four acceleration vector of congruence of the timelike geodesics is zero [66, 72]. Now, using Eq. (33) to get:

$$\frac{d\Theta}{d\lambda} + \frac{1}{3}\Theta^2 + \sigma_{\mu\nu}\sigma^{\mu\nu} - \omega_{\mu\nu}\omega^{\mu\nu} + 8\pi G_N(\tau'_{\mu\nu} - \frac{1}{2}\tau' q_{\mu\nu})u^\mu u^\nu = -\frac{1}{12}\kappa_5^2(\rho(2\rho + 3p)). \quad (44)$$

It is also easy to show that the right hand side of this equation can be written as:

$$\frac{1}{12}(\rho(2\rho + 3p)) = (\Pi_{\mu\nu} - \frac{1}{2}\Pi q_{\mu\nu})u^\mu u^\nu. \quad (45)$$

By combining Eqs. (44) and (45), and using the Einstein equations onto the brane (30) we get

$$\frac{d\Theta}{d\lambda} = -\frac{1}{3}\Theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - {}^4R_{\mu\nu}u^\mu u^\nu. \quad (46)$$

It is apparent that when $\kappa_5 \rightarrow 0$, while $G_N \rightarrow c \neq 0$, this equation converges to that of the Einstein theory [60, 67]. Since for the hypersurface orthogonal congruences $\omega_{\mu\nu} = 0$, just the same as the GR, the attractive nature of the gravity implies ${}^4R_{\mu\nu}u^\mu u^\nu \geq 0$ [66]. In fact, it is shown that for a general situation, including $E_{\mu\nu} \neq 0$ and arbitrary source of energy, ${}^4R_{\mu\nu}u^\mu u^\nu \geq 0$ leading to $({}^4T_{\mu\nu} - \frac{1}{2}{}^4T q_{\mu\nu})u^\mu u^\nu \geq 0$ can be considered as a suitable criterion for studying SEC [66]. It is also useful to mention here that the $E_{\mu\nu}$ term may lead to violate SEC [71]. More comprehensive notes about the energy conditions as well as the matter evolution in this theory can be found in [66, 67, 71].

In order to calculate the induced Komar mass on the brane, bearing the traceless nature of $E_{\beta\delta}$ in mind. In addition, we insert $n = 3$ into Eq. (18), after using Eq. (31), we reach at

$$\mathcal{M} = 2 \int_{V_3} dV_3 \left[\left(\tau'_{\mu\nu} + \frac{6}{\lambda}\Pi_{\mu\nu} - \frac{6}{\lambda\kappa_5^2}E_{\mu\nu} \right) - \frac{1}{2} \left(\tau' + \frac{6}{\lambda}\Pi \right) q_{\mu\nu} \right] n^\mu \xi^\nu. \quad (47)$$

In this equation, n^μ and ξ^ν are the normal unit vector to the volume V_3 and the timelike Killing vector on the brane, respectively. It is useful to mention that for the positive values of RHS of this equation we get $\mathcal{M} \geq 0$ which is similar to ASEC [60]. From Eq. (23) we see that $E_{\mu\nu}$ vanishes either for the flat brane spacetime ($q_{\mu\nu}$) or the Ads bulk [56, 66]. This leads us to

$$\mathcal{M} = 2 \int_{V_3} dV_3 \left[\left(\tau'_{\mu\nu} + \frac{6}{\lambda}\Pi_{\mu\nu} \right) - \frac{1}{2} \left(\tau' + \frac{6}{\lambda}\Pi \right) q_{\mu\nu} \right] n^\mu \xi^\nu. \quad (48)$$

Such condition ($E_{\mu\nu} = 0$) leads to $\partial_\mu T^{\mu\nu} = 0$, implying that the energy-momentum conservation law on the brane is the same as that of the Einstein gravity only for the AdS₅ bulk [56]. Assuming the Randall-Sundrum fine-tuning, $Q = \frac{3\Lambda}{\kappa_5\lambda} + \frac{\lambda}{2} = 0$, holds on the brane one can easily check that

$$\mathcal{M} = 2 \int_{V_3} dV_3 \left[\left(\tau_{\mu\nu} + \frac{6}{\lambda} \Pi_{\mu\nu} \right) - \frac{1}{2} \left(\tau + \frac{6}{\lambda} \Pi \right) q_{\mu\nu} \right] n^\mu \xi^\nu, \quad (49)$$

which is compatible with the results obtained in [42, 43]. Therefore, unlike Eq. (47), the Komar mass definition used in [42, 43] is confined to the brane satisfying the $E_{\mu\nu} = 0$ and $Q = 0$ conditions simultaneously. Finally, we should note that our relation for the Komar mass (47) is more comprehensive than Eq. (49) introduced in [42, 43]. Equating Komar definition for the mass in (49) with Eq. (17) we finally obtain

$$G_{\mu\nu} = 8\pi G \left(\tau_{\mu\nu} + \frac{6}{\lambda} \Pi_{\mu\nu} \right). \quad (50)$$

Bearing $E_{\mu\nu} = 0$ and $Q = 0$ in mind, we see that this equation is similar to the Einstein field equations on the brane (30), provided we take $G = G_N$ yielding $\Lambda = -\frac{4}{3\ell_p}$. The negative sign in $\Lambda = -\frac{4}{3\ell_p}$ is signalling that the bulk spacetime should be AdS. Indeed, comparing this equation with Eq. (30), we find that Eq. (50) is valid if the bulk spacetime is AdS. Finally, We should also note that the Komar mass definition (49), introduced in [42, 43], along as the entropic force scenario cannot lead to the true Einstein field equations on the brane.

III. CONCLUSIONS AND DISCUSSIONS

We have considered the higher dimensional definition for the Komar mass, and by taking into account the entropic origin for gravity, we derive the Einstein field equations in arbitrary dimensions. This procedure naturally leads to the derivation of the higher dimensional gravitational coupling constant of the Einstein equations which is in complete agreement with the results obtained by comparing the weak field limit of the Einstein equations with Poisson equation in higher dimensions [48, 62]. We mentioned that one can find the Einstein equations onto the brane by reflecting the Einstein equations on the $(3+1)$ -dimensional submanifold. The quality of availability of some energy conditions, including WEC, NEC, DEC and SEC, were briefly studied. Then, we discussed the differences between Newtonian gravitational constant on the brane, G_N , and the ordinary Newtonian gravitational constant (G). This difference originates from the higher dimensional considerations and projective nature of the Einstein equation on the brane. In addition, we have derived an expression for the Komar mass on the brane, and compared our relation with that of used in previous works [42, 43]. Finally, by considering the entropic origin for the gravity and employing the Komar definition for mass [42, 43], we found the Einstein field equations onto the 3-brane embedded in a five dimensional AdS bulk. In addition, we pointed to the weaknesses of the Einstein field equations on the brane when the Komar mass definition introduced in [42, 43] is used.

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- [1] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
 - [2] S. W. Hawking., Commun Math. Phys. **43**, 199 (1975).
 - [3] S. W. Hawking., Nature **248**, 30 (1974).
 - [4] J. M. Bardeen, B. Carter, S. W. Hawking, Commun. Math. Phys. **31**, 161 (1973).
 - [5] P. C. W. Davies, J. Phys. A: Math. Gen. **8**, 609 (1975).
 - [6] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
 - [7] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
 - [8] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).
 - [9] C. Eling, R. Guedens, T. Jacobson, Phys. Rev. Lett. **96**, 121301 (2006).
 - [10] T. Padmanabhan, Class. Quantum. Grav. **19**, 5387 (2002).

- [11] M. Akbar, R. G. Cai, Phys. Rev. D **75**, 084003 (2007).
- [12] R. G. Cai, L. M. Cao, Phys. Rev. D **75**, 064008 (2007).
- [13] R. G. Cai, S. P. Kim, JHEP **0502**, 050 (2005).
- [14] B. Wang, E. Abdalla, K. R. Su, Phys. Lett. B **503**, 394 (2001).
- [15] B. Wang, E. Abdalla, K. R. Su, Mod. Phys. Lett. A **17**, 23 (2002).
- [16] R. G. Cai, M. L. Cao, P. Y. Hu, JHEP **0808**, 090, (2008).
- [17] S. Nojiri, S. D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006).
- [18] A. Sheykhi, Class. Quantum. Grav. **27**, 025007 (2010).
- [19] A. Sheykhi, Eur. Phys. J. C **69**, 265 (2010).
- [20] A. Sheykhi, B. Wang, R. G. Cai, Nucl. Phys. B **779**, 1 (2007).
- [21] R. G. Cai, L. M. Cao, Nucl. Phys. B **785**, 135 (2007).
- [22] A. Sheykhi, B. Wang, R. G. Cai, Phys. Rev. D **76**, 023515 (2007).
- [23] A. Sheykhi, B. Wang, Phys. Lett. B **678**, 434 (2009).
- [24] A. Sheykhi, JCAP **05**, 019 (2009).
- [25] T. Padmanabhan, Rept. Prog. Phys. **73**, 046901 (2010).
- [26] T. Padmanabhan, Mod. Phys. Lett. A **25**, 1129 (2010).
- [27] E. Verlinde, JHEP **1104**, 029 (2011).
- [28] R. G. Cai, L. M. Cao, N. Ohta, Phys. Rev. D **81**, 061501(R) (2010).
- [29] R. G. Cai, L. M. Cao, N. Ohta, Phys. Rev. D **81**, 084012 (2010).
- [30] L. Smolin, arXiv:1001.3668.
- [31] M. Li, Y. Wang, Phys. Lett. B **687**, 243 (2010).
- [32] Y. Tian, X. Wu, Phys. Rev. D **81**, 104013 (2010).
- [33] Y. S. Myung, arXiv:1002.0871.
- [34] I. V. Vancea, M. A. Santos, arXiv:1002.2454.
- [35] L. Modesto, A. Randono, arXiv:1003.1998.
- [36] A. Sheykhi, Phys. Rev. D **81**, 104011 (2010).
- [37] B. Liu, Y. C. Dai, X. R. Hu, J. B. Deng, Mod. Phys. Lett. A **26**, 489 (2011).
- [38] S. H. Hendi, A. Sheykhi, Phys. Rev. D **83**, 084012 (2011).
- [39] A. Sheykhi, S. H. Hendi, Phys. Rev. D **84**, 044023 (2011).
- [40] S. H. Hendi, A. Sheykhi, Int. J. Theor. Phys. **51**, 1125 (2012).
- [41] A. Sheykhi, Z. Teimoori, Gen. Relativ. Gravit. **44**, 1129 (2012).
- [42] Y. Ling, J. P. Wu, J. Cosmol. Astropart. Phys. **1008**, 017 (2010).
- [43] A. Sheykhi, Int. J. Theor. Phys. **51**, 185 (2012).
- [44] W. Gu, M. Li, R. X. Miao, arXiv:1011.3419.
- [45] R. X. Miao, J. Meng, M. Li, arXiv:1102.1166.
- [46] Y. X. Liu, Y. Q. Wang, S. W. Wei, Class. Quantum Grav. **27**, 185002 (2010).
- [47] R. B. Mann, J. R. Mureika, Phys. Lett. B **703**, 167 (2011).
- [48] A. Sheykhi, H. Moradpour, N. Riazi, Gen. Rel. Grav. **45**, 1033 (2013).
- [49] J. Polchinski, *String Theory I & II* (Cambridge Univ. Press, Cambridge, 1998).
- [50] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
- [51] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999).
- [52] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
- [53] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002).
- [54] L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [55] L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
- [56] T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
- [57] A. Paranjape, S. Sarkar and T. Padmanabhan, Phys. Rev. D **74**, 104015 (2006).
- [58] T. Padmanabhan, Phys. Rev. D **81**, 124040 (2010).
- [59] R. M. Wald, *General Relativity* (The University of Chicago Press, 1984).
- [60] E. Curiel, arXiv:1405.0403.
- [61] T. Padmanabhan, Class. Quantum. Grav. **21**, 4485 (2004).
- [62] R. Mansouri, A. Nayeri, Gravitation Cosmology, **4**, No. 2, 142 (1998).
- [63] O. Gron and S. Hervik, *Einstein's General Theory of Relativity* (Springer LLC, 2007).
- [64] G. F. R. Ellis, R. Martens and M. A. H. Maccallum, *Relativistic Cosmology* (Cambridge Univ. Press, New York, 2012).
- [65] U. Ellwanger, Mod. Phys. Lett. A **20**, 2521 (2005).
- [66] R. Martens, Living Rev. Rel. **7**, 7 (2004).
- [67] R. Martens, Phys. Rev. D **62**, 084023 (2000).
- [68] M. Sasaki, T. Shiromizu, and K. Maeda, Phys. Rev. D **62**, 024008 (2000).
- [69] E. Poisson, *A Relativist's Toolkit* (Cambridge University Press, UK, 2004).
- [70] S. W. Hawking and G. F. R. Ellis, *the large scale structure of space-time*, (Cambridge University Press, USA, 1973).
- [71] D. N. Vollick, Gen. Rel. Grav. **34**, 1 (2002).
- [72] G. F. R. Ellis, and H. van Elst, NATO Adv. Study Inst. Ser. C. Math. Phys. Sci. **541**, 1 (1999) [arXiv:gr-qc/9812046].